

FUNDAMENTAL AND STATISTICAL MODELING

- 8.1. Application of Factorial Experiment Design in the Measurement of Drying Rates of Rotary Dryer
- 8.2. Change of Viscosity with Respect to Temperature and Concentration
- 8.3. Modeling of Water Level in a Tank under Steady and Dynamic Conditions

8.1. APPLICATION OF FACTORIAL EXPERIMENT DESIGN IN THE MEASUREMENT OF DRYING RATES OF ROTARY DRYER

Keywords: *Drying, experimental design, full factorial, factorial design, rotary drying.*

Before the experiment: *See your TA for the preparation of wet solids ONE DAY BEFORE THE EXPERIMENT!!!*

8.1.1. Aim

To investigate how various operational parameters affect the drying process in a rotary dryer using factorial experimental design.

8.1.2. Theory

Drying is a physical separation process that has as its objective the removal of a liquid phase by means of thermal energy. The liquid, in most cases water, is liberated by the process of vaporization. The dry solid will usually contain an amount of residual moisture. Dryers, used for this purpose, may be classified according to the physical characteristics of the material being dried, the method of transferring the thermal energy to the wet solid, the source of the thermal energy, the method of the physical removal of the solvent vapor, and the method of dispersion or mixing (agitation) of the wet solid in the drying operation.

Rotary dryers are suitable for free-flowing non-sticking materials of relatively small particle size. When the materials to be dried can be safely brought into contact with the heating medium - generally air - and are not too dusty, direct rotary dryers are used [3]. A direct rotary dryer consists of a rotating cylindrical shell, slightly inclined to the horizontal, through which the heating medium flows in a co-current or in a countercurrent direction to the flow of the wet medium. It is usually equipped with "flights" on the interior for lifting and shoveling the wet solids through the gas stream during passage through the cylinder. The movement of the material is due to the combined effect of the inclination of the dryer and the action of internal lifting flights [3, 4].

Progress of material through a direct rotary dryer, i.e., retention or residence time, is affected by nine factors, namely: (1) Percentage loading within the dryer; (2) number of flights distributed along the circumference; (3) design of the flights; (4) slope of the dryer from the

horizontal; (5) rate of rotation of the dryer shell; (6) length of the dryer effectively used; (7) diameter of the dryer; (8) physical properties of the material; (9) air velocity within the dryer [4] .

Design of experiments (DOE) is a systematic approach to determine the relationship between factors affecting a process and the output of that process. In other words, it is used to establish a cause-and-effect relationship between a number of independent variables and a dependent variable of interest. In order to optimize the process output, one should be able to manage the process inputs using this cause-and-effect relationship information. The dependent variable, is called the **response**, and the independent variables which are effecting the dependent variable are called **factors**. Different factor values at which experiments are performed, are called **levels**.

Randomization refers to the order in which the trials of an experiment are performed. Performing the experiments with a randomized sequence eliminates the effects of unknown or uncontrolled variables. Randomization is the most reliable and plausible method to obtain homogenous observations without involving any potential biases or judgements.

When more than one factor is of interest in an experiment, a factorial experiment should be used. Factorial design mean that all possible combinations of the levels of the factors are investigated.

The two factor factorial design is the simplest types of factorial design, which involve only two factors or sets of treatments. Suppose that there are “*a*” levels of factor A, “*b*” levels of factor B, and measurements are repeated for “*n*” times. The general experimental data points are arranged as:

		Factor B				
		1	2	...	b	
Factor A	1	$y_{111}, y_{112}, \dots, y_{11n}$ $(\bar{y}_{11.})$	$y_{121}, y_{122}, \dots, y_{12n}$ $(\bar{y}_{12.})$		$y_{1b1}, y_{1b2}, \dots, y_{1bn}$ $(\bar{y}_{1b.})$	$(\bar{y}_{1..})$
	2	$y_{211}, y_{212}, \dots, y_{21n}$ $(\bar{y}_{21.})$			$y_{2b1}, y_{2b2}, \dots, y_{2bn}$ $(\bar{y}_{2b.})$	$(\bar{y}_{2..})$
	...					
	a	$y_{a11}, y_{a12}, \dots, y_{a1n}$ $(\bar{y}_{a1.})$	$y_{a21}, y_{a22}, \dots, y_{a2n}$ $(\bar{y}_{a2.})$		$y_{ab1}, y_{ab2}, \dots, y_{abn}$ $(\bar{y}_{ab.})$	$(\bar{y}_{a..})$
		$(\bar{y}_{.1.})$	$(\bar{y}_{.2.})$		$(\bar{y}_{.b.})$	$(\bar{y}_{..})$

Let $y_{i..}$ denote the total of all observations under the i th level of factor A, $y_{.j.}$ denote the total of all observations under the j th level of factor B, $y_{ij.}$ denote the total of all observations in the ij th cell, and $y_{...}$ denote the grand total of all observations. Define $\bar{y}_{i..}, \bar{y}_{.j.}, \bar{y}_{ij.}, \bar{y}_{...}$ as the corresponding row, column, cell and grand averages.

These observations may be described by a linear statistical model

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \\ k = 1, 2, \dots, n \end{cases}$$

where y_{ijk} is the observed response (taken during experiment) when factor A is at the i th level ($i = 1, 2, \dots, a$) and factor B is at the j th level ($j = 1, 2, \dots, b$) for the k th replicate ($k = 1, 2, \dots, n$).

Here, $\hat{\tau}_i = \bar{y}_{i..} - \bar{y}_{...}$ is equal to the fixed effect of i th level of row, $\hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...}$ is equal to the fixed effect of j th level of column, and $\widehat{(\tau\beta)}_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$ is the fixed effect of ij th level of cell.

Estimate of each observation can be computed as

$$\hat{y}_{ijk} = \bar{y}_{...} + \hat{\tau}_i + \hat{\beta}_j + \widehat{(\tau\beta)}_{ij}$$

The residuals, which are simply the difference between the observations and the corresponding cell averages, are calculated as follows:

$$e_{ijk} = y_{ijk} - \hat{y}_{ijk}$$

Analysis of variance (ANOVA) is an analysis tool used to divide the total variability found inside a data set into two parts: systematic factors and random factors. The systematic factors are the factors that have a statistical influence on the given data set, while the random factors do not have a statistical effect on the given data set.

If we suppose that $y_{111}, y_{112}, \dots, y_{a11}, \dots, y_{ab1}, \dots, y_{abn}$ is a random sample from an $N(\mu, \sigma^2)$ distribution. Then SS/σ^2 is distributed as chi-square with $n - 1$ degrees of freedom.

$$\frac{SS}{\sigma^2} = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{\sigma^2} \sim \chi_{n-1}^2$$

Notice that the total sum of squares has been partitioned into a sum of squares due to “rows” or factor A, (SS_A); a sum of square due to “columns” or factor B, (SS_B); a sum of squares due to the interaction between A and B, (SS_{AB}); and a sum of squares due to error (SS_E). Expressed mathematically,

The sums of squares for the main effects are

$$SS_A = bn \sum_{i=1}^a (\bar{y}_{i..} - \bar{y}_{...})^2 \quad SS_B = an \sum_{j=1}^b (\bar{y}_{.j.} - \bar{y}_{...})^2$$

The sums of squares for the interaction term is

$$SS_{AB} = n \sum_{i=1}^a \sum_{j=1}^b (\bar{y}_{ij.} - \bar{y}_{...})^2 - SS_A - SS_B$$

At the end, we may compute SS_E and SS_T as

$$SS_E = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n (\bar{y}_{ijk} - \bar{y}_{ij.})^2$$

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E$$

The numbers of degrees of freedom associated with each sum of square are $a - 1$, $b - 1$, $(a - 1)(b - 1)$, $ab(n - 1)$ for the effects A, B, AB interaction and error respectively.

Each sum of squares divided by its degrees of freedom is a mean square.

$$MS_A = \frac{SS_A}{a - 1}, \quad MS_B = \frac{SS_B}{b - 1}, \quad MS_{AB} = \frac{SS_{AB}}{(a - 1)(b - 1)}, \quad MS_E = \frac{SS_E}{ab(n - 1)}$$

Then , F test is the simply ratio between groups variance estimate to the within groups variance estimate, and F-values are calculated for A treatments, B treatments and interaction as

$$F_0 = MS_A/MS_E, F_0 = MS_B/MS_E \text{ and } F_0 = MS_{AB}/MS_E, \text{ respectively.}$$

ANOVA TABLE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	SS_A	$a - 1$	MS_A	MS_A/MS_E
B	SS_B	$b - 1$	MS_B	MS_B/MS_E
Interaction	SS_{AB}	$(a - 1)(b - 1)$	MS_{AB}	MS_{AB}/MS_E
Error	SS_E	$ab(n - 1)$	MS_E	
Total	SS_T	$abn - 1$		

The hypotheses that we will test are as follows:

$$H_0: \tau_1 = \tau_2 = \dots = \tau_a = 0 \text{ (no main effect of factor A)}$$

$$H_1: \tau_1 \text{ at least one } \tau_i \neq 0$$

This null hypothesis is rejected at α level of significance if $f_0 > f_{\alpha, a-1, ab(n-1)}$.

$$H_0: \beta_1 = \beta_2 = \dots = \beta_b = 0 \text{ (no main effect of factor B)}$$

$$H_1: \beta_1 \text{ at least one } \beta_i \neq 0$$

This null hypothesis is rejected at α level of significance if $f_0 > f_{\alpha, b-1, ab(n-1)}$.

$$H_0: (\tau\beta)_{11} = (\tau\beta)_{12} = \dots = (\tau\beta)_{ab} = 0 \text{ (no interaction)}$$

$$H_1: \tau_1 \text{ at least one } (\tau\beta)_{ij} \neq 0$$

This null hypothesis is rejected at α level of significance if $f_0 > f_{\alpha, (a-1)(b-1), ab(n-1)}$

To check the model adequacy, the residuals should be normally distributed, and the residuals plot should not show any trend.

8.1.3. Experimental Setup

The experimental setup used in this experiment is shown in Figure 7.1.1. The dimensions of the equipment are: Length 125 cm and Diameter 25 cm.

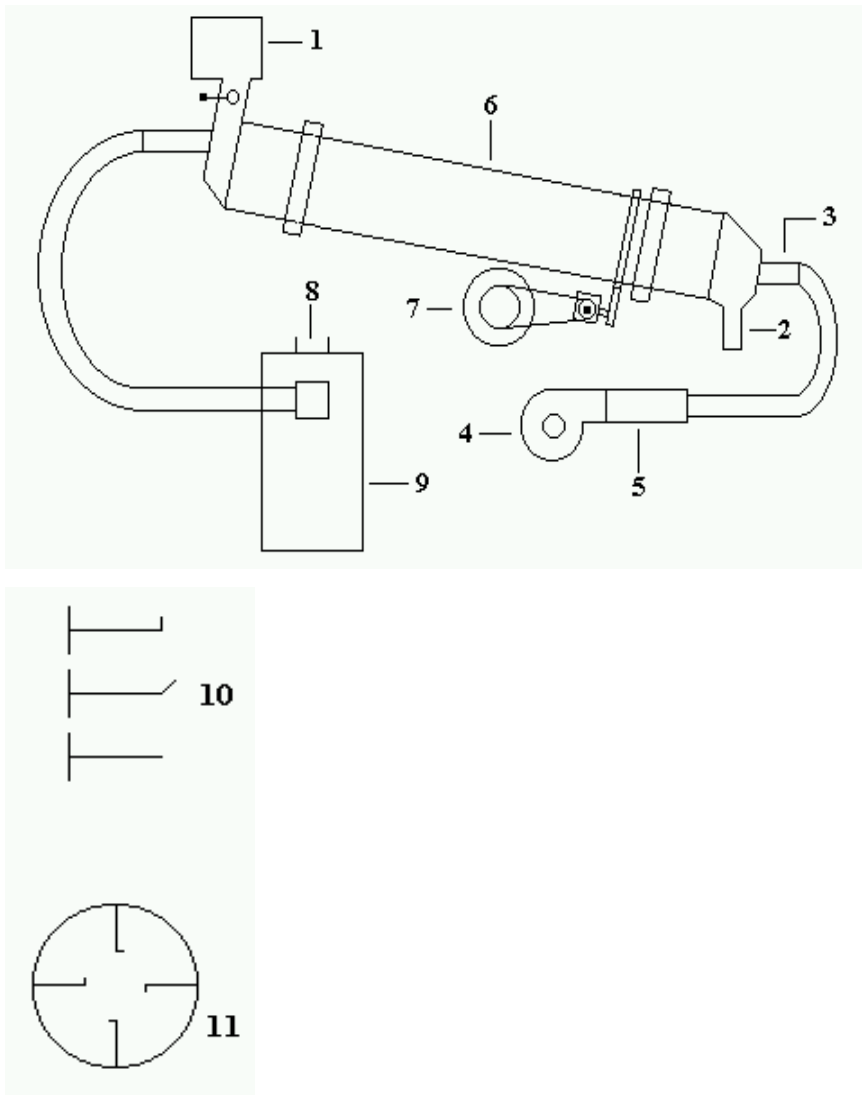


Figure 8.1.1. Direct rotary dryer.

- | | |
|----------------------------|---|
| 1.) Wet Material Inlet | 7.) Electrical Drive |
| 2.) Dry Material Discharge | 8.) Air Outlet |
| 3.) Air Inlet | 9.) Cyclone |
| 4.) Air Fan | 10.) Types of Flights Used in the Shell |
| 5.) Air Heater | 11.) Cross-Section of the Shell |
| 6.) Cylindrical Shell | |

8.1.4. Procedure

1. Weigh about 1 kg of chickpea and divide solids into 2 batches of 0.5 kg. Wet the first batch of solid with water **a day prior** to the experiment. Wet the second batch of solid with water **1 hours prior** to the experiment.
2. Determine the humidity of the two batches using an oven (1 hr or more at 105°C) by weighing samples from the both batches before and after heat treatment (drying in oven).
3. Divide both batches into 6 samples (12 samples in total).
4. Choose a rate of rotation for the dryer shell.
5. Pour the first wet solid sample into the feeder of the rotary dryer.
6. Collect the discharged product in plastic containers and weigh them.
7. Determine the humidity of the discharged product by using an oven (1 hr or more at 105°C).
8. Repeat the procedure steps 4-7 for three different rates of rotation for both batches (in total 6 runs will be made).
9. Repeat 4-8.

NOTE: Conduct the experiments in **random order**.

Safety Issues: This experiment does not involve use of any hazardous or corrosive chemicals. The rotary dryer unit operates with hot air; hence the outer shell of the unit will be hot, therefore avoid any skin contact with the outer shell. At all times wear lab coats, eyewear and gloves.

8.1.5. Report Objectives

Suppose that the response variable y_{ijk} is defined as the ratio of difference in moisture content before and after evaporation to initial moisture content, which is equal to the ratio of amount of water to dry weight of chickpeas. Define Factor A as the wetting time, and Factor B as revolution.

1. Determine \widehat{y}_{ijk} and calculate residuals for 12 observations.
2. Give normal probability plot (via '*normplot*' in MATLAB) of residuals.
3. Calculate sum of squares, mean square errors and F-values, and construct ANOVA table completely.

4. Calculate critical F-values and determine whether A, B treatments and AB interaction are significant or not.
5. Discuss the result of residuals and their normal probability plot.
6. Discuss significance of revolution and wetting time and their interaction on evaporation in rotary dryer.

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8.2. CHANGE OF VISCOSITY WITH RESPECT TO TEMPERATURE AND CONCENTRATION

Keywords: Viscosity, temperature, mole fraction, binary liquid mixture, falling ball viscometer, heating circulator

Before the experiment: Read the booklet carefully. Be aware of the safety precautions.

8.2.1. Aim

To measure the viscosity of an alcohol/water mixture in falling ball viscometer and to investigate the effect of temperature and the mole fraction of the mixture on the viscosity.

8.2.2. Theory

Viscosity is the property of resistance to flow and can be considered as the friction between the molecules of fluids. In mathematical aspect, it is the ratio of tangential frictional force per unit area to the velocity gradient normal to the flow direction. [1] There are three well known methods to measure the viscosity: (1) moving surface in a contact with fluid, (2) fluid flow against a resistance and (3) an object moving through a fluid. [2]

8.2.2.1. Dynamic Viscosity

Falling ball viscometer is used for measuring the viscosity of Newtonian liquids and gases. The working principle of the apparatus is based upon Newton's law of motion for a sphere falling ball at its terminal velocity. In the Newton's law of motion for a falling ball, there exists buoyancy force, F_B weight force, W and drag force, F_D as well as these three forces reach net force of zero as below. [2]

$$W - F_B - F_D = \left(\frac{\pi d^3}{6}\right)\gamma_s - \left(\frac{\pi d^3}{6}\right)\gamma_f - 3\pi\mu u_t d = 0 \quad (8.2.1)$$

where,

d : diameter of sphere, m

u_t : terminal velocity,	m/s
γ_s : specific weight of sphere,	N/m ³
γ_f : specific weight of fluid	N/m ³
μ : (dynamic) viscosity of the fluid,	N·s/m ² (poise, P)

At low Re, i.e. $Re \ll 1$, the drag force can be expressed as in the third term on the right side of the equal sign. The Eqn. 8.2.1 can be arranged as

$$\mu = \frac{(\gamma_s - \gamma_f)d^2}{18u_t} \quad (8.2.2)$$

The above equation can also be simplified as below.

$$\mu = K(\rho_s - \rho_f)t \quad (8.2.3)$$

where

$$K = \frac{g \cdot d^2}{18l} \quad (8.2.4)$$

K : ball constant	Pa·m ³ /kg
ρ_s : mass density of the sphere,	kg/m ³
ρ_f : mass density of the fluid,	kg/m ³
l : falling length,	m
g : gravitational acceleration,	m/s ²
t : time passing the length of l	s

The Eqn. 8.2.3 clearly shows that for falling tube systems, the (dynamic) viscosity can simply be calculated by measuring the falling time of sphere ball in cylindrical tube at a length l if the density of both the fluid and sphere and the ball constant.

8.2.2.2. Kinematic Viscosity

Kinematic viscosity is a physical quantity which can be obtained by dividing (dynamic) viscosity with the density of the fluid, [3] as shown below:

$$\nu = \frac{\mu}{\rho} \quad (8.2.5)$$

where,

ν : kinematic viscosity of the fluid, m^2/s (stoke, St)

8.2.2.3. Effect of Temperature

The viscosities of the fluids dominantly depend on molecular structure and temperature. The relationship between temperature and viscosity is inversely proportional for all fluids. As the temperature of the fluid increases, the mean free path of the molecules traveled decreases since the increase in thermal velocity causes more molecules move around and interact to each other. The decrease in viscosity with increasing temperature can be represented in analogous to Arrhenius equation:

$$\frac{1}{\mu} = C e^{\frac{-E}{RT}} \quad (8.2.6)$$

Where R is universal gas constant, C is a system-dependent constant and E is the activation energy required to overcome the internal friction. The activation energy is determined from the slope of $\ln \mu$ vs $1/T$ plot through linearization of equation 8.2.2. [4]

8.2.2.4. Effect of Mole Fraction of Binary Liquid Mixture

For a liquid mixture, the difference between measured viscosity values is sourced by the non-ideal behavior of the liquids. This phenomena is generally associated with the volume contraction and mixing enthalpy. [4]

Alcohol molecules in aqueous solutions give strong influence on the structure of water molecules and the presence of alcohol may result in certain anomalies in various physical properties. [5] For example, at microscopic level, the water and methanol molecules mix very little due to the open-ring structure formation of liquid methanol molecules in the presence of water. [6] The viscosity of alcohol/water solutions differs from other binary liquid mixtures. Methanol/water solution has a maximum viscosity at a specific mixing ratio, in which the viscosity is bigger than both methanol and water.

The viscosity of the methanol/water solution is measured and the change in molar ratio is investigated in many studies [7, 8] and mostly a polynomial empirical equations (see Eqn. 8.2.7) are established. [7]

$$\mu = \sum_{i=0}^n a_i x_1^i \quad (8.2.7)$$

where,

n : polynomial degree,

i : degree of a polynomial term,

a : coefficient,

x_1 : mole fraction of one of the liquids

8.2.3. Experimental Setup

The schematic diagram of the experimental setup is shown in Figure 8.2.1.

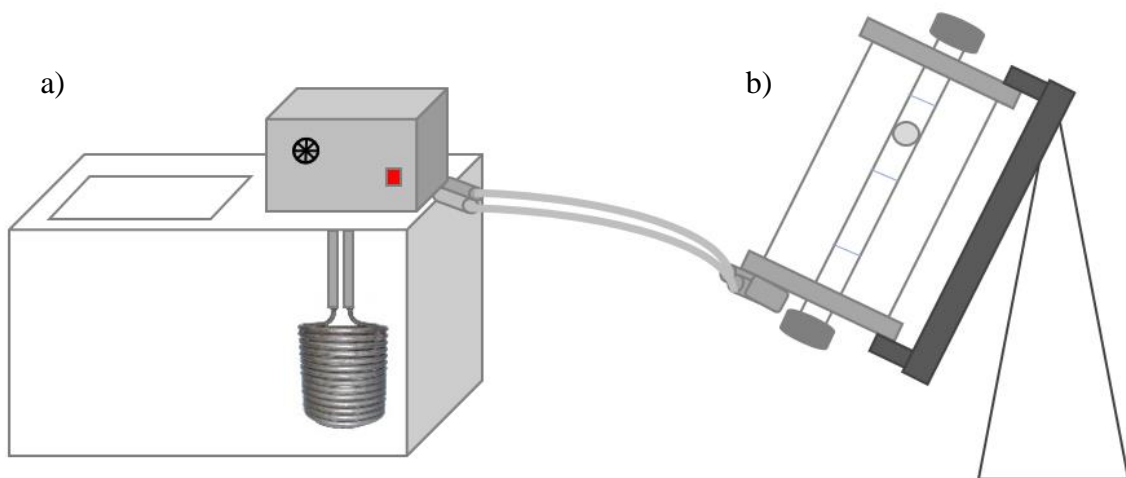


Figure 8.2.1. Experimental setup including a) a circulating thermostat and b) a falling ball viscometer.

For the experiment, a stopwatch to measure the falling time of the ball, a thermometer to determine the temperature of the tubular jacket, distilled water, methanol and 50 mL volumetric flasks to prepare the liquid mixtures are also needed.

8.2.4. Procedure

1. Connect the falling ball viscometer to the pump connection unit of the thermostat with tubing.
2. Fill the bath of the circulating thermostat with distilled water.
3. Calibrate the falling ball viscometer and fill it bubble-free with the liquid to be investigated (water, methanol and methanol-water mixtures).
4. Place the ball, which is appropriate for the viscosity range of the investigated liquid, in the gravity tube and allow the viscometer to equilibrate to selected temperature, T for 10 minutes.
5. Determine two falling times t and calculate the arithmetic mean of the measured values.
6. Repeat the experiment for all test mixtures.
7. Measure the temperature dependence of the viscosity of pure liquids in the temperature range between 298 K and 323 K.

8.2.5. Report Objectives

1. Calculate the dynamic and kinematic viscosities of water, methanol and methanol-water mixtures.
2. Plot the viscosity values in terms of mole fraction of the liquids at 298 K.
3. Plot the viscosity values of pure substances against different temperatures.
4. Calculate the energy barriers of water and methanol.
5. Establish an empirical model by a power series for dynamic viscosity as a function of mole fraction of methanol.

Safety Issues: Methanol is highly flammable and an acute toxin. Methanol and methanol-water mixtures are hazardous in case of skin contact, eye contact, ingestion and inhalation. Do not ingest, breathe and avoid contact with skin and eyes. Keep away from incompatibles such as oxidizing agents, metals and acids.

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Appendix

Table A.1. Parameters of Sphere Balls [10].

Ball No.	Viscosity measurement range (guideline value) (mPa•s)	Material (Recommendation)	Density (guideline value) (g/cm ³)	Ball diameter (mm)	Calibration ball constant (guideline value) (mPa•s•cm ³ /g•s)
1	0.5 to 10	Borosilicate glass	2.4	15.81 ± .01	0.007
2	9 to 140	Borosilicate glass	2.4	15.6 ± 0.05	0.09
3	40 to 700	Ni/iron	8.1	15.6 ± 0.05	0.09
4	150 to 5,000	Ni/iron	8.1	15.2 ± 0.1	0.7
5	1,500 to 50,000	Ni/iron	8.1	14.0 ± 0.5	7
6	Above 7,500	Ni/iron	8.1	11.0 ± 1	35

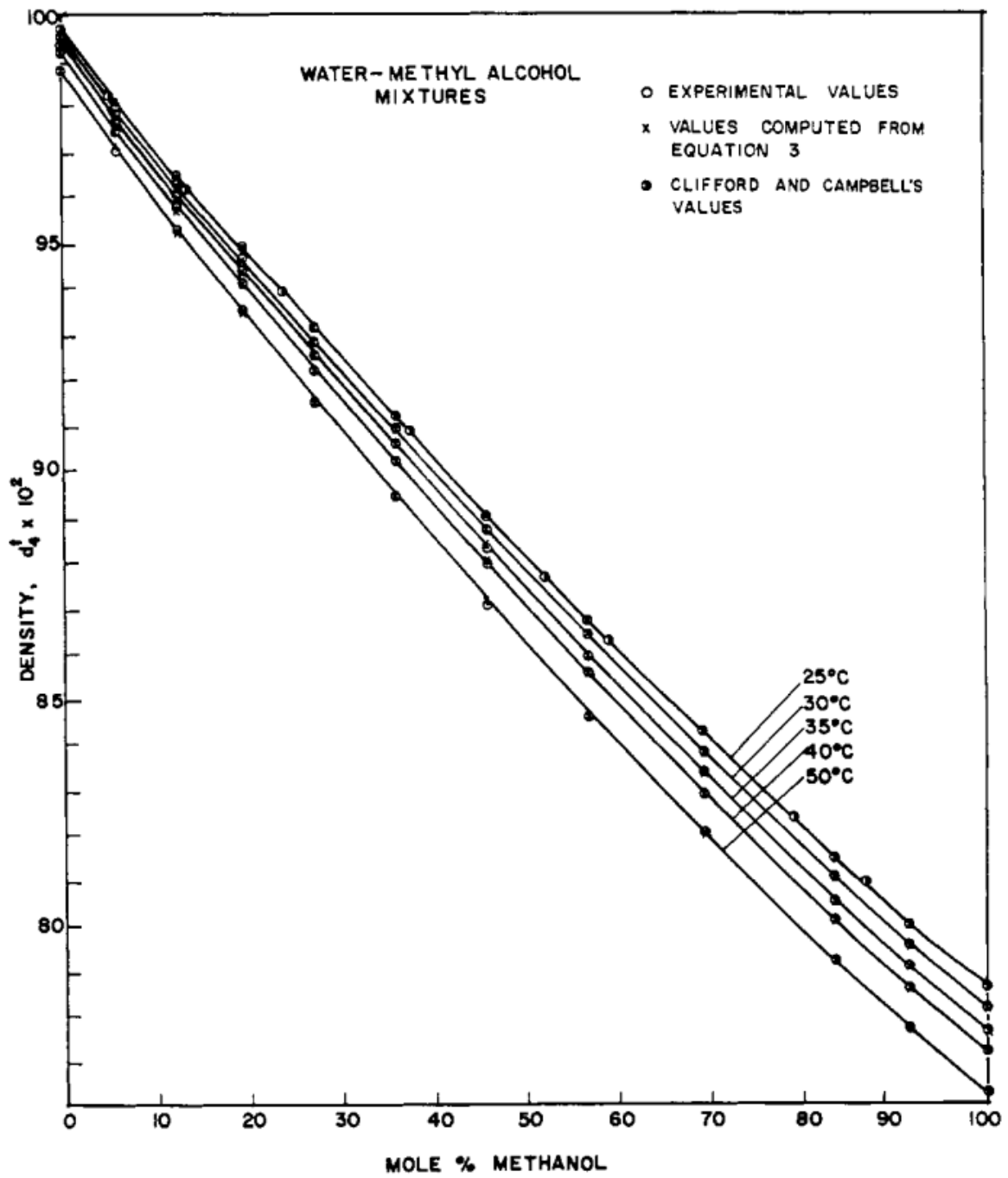


Figure A.1. Experimental data for density vs. mole percent methanol. [7]

8.3. LINEAR AND NONLINEAR MATHEMATICAL MODELS OF WATER LEVEL IN A TANK

Keywords: *Mathematical model, differential equation, linear vs. nonlinear, system, water tank*

8.3.1. Object

The object of this experiment is to propose linear and nonlinear models for describing the dynamic behavior of water level in a tank and then simulate these two models for comparing with the experimental data and in between each other.

8.3.2. Theory

8.3.2.1. Modeling

Modeling is the art of creating mathematical descriptions of, e.g. physical, chemical or electrotechnical phenomena which appear in reality. These descriptions have to be relatively simple, yet accurate enough to serve the purpose of the modeler [1].

It is important to realize that many different models exist, all describing some different parts of the same reality. It depends on the point of view and intention of the modeler which part of that reality is described. In studying a chemical process, the modeler may be interested in the chemical reactions, in the physical working conditions, in the mechanical construction of the reactor, in the possible environmental impact of the reactor, in the financial return on investments and in the dynamic behavior of the process [2]. All these different views will yield different models.

The essence of the art of modeling is to select only those characteristics, from among that are necessary and sufficient to describe the process accurately enough to suit the objects of the modeler. The simplified models allow the modeler to grasp the essentials from a turbulent and sometimes chaotic world [1]. So, modeling is grasping the central issue from reality and translating it into an abstract language such as a mathematical model. An important decision in deriving models is the selection of the *system boundary*. The isolated part of the process will be

called the *system* and all the parts of reality not belonging to the system are attributed to the *environment* of the system [1].

8.3.2.2. Differential Equations

It is often the case that the mathematical models used for describing the systems involve an equation in which a function and its derivatives play important roles. Such equations are called differential equations [3].

$$\frac{dx}{dt} = k_1x + k_2 \quad (8.3.1)$$

$$\frac{dy}{dx} = F(x)x + y \quad (8.3.2)$$

$$\frac{dx}{dt} = Ax^2 \quad (8.3.3)$$

When an equation involves one or more derivatives with respect to a particular variable, that variable is called an independent variable. A variable is called dependent if a derivative of that variable occurs in the equation [3]. In Equation 8.3.1, x is the dependent and t is the independent variable. When t is the independent variable, the following notation may be used:

$$x' = Ax^2 \quad (8.3.4)$$

where $x' = \frac{dx}{dt}$.

The *order* of a differential equation is the order of the highest ordered derivative appearing in the equation, whereas the *degree* of a differential equation is the power to which the highest derivative is raised when the equation has been rationalized. For instance,

$$y'' + 2cy' + y = 0 \quad (8.3.5)$$

is an equation of order two (or a second order equation) and first degree.

Differential equation can be classified as *linear* and *nonlinear*. A linear differential equation is one for which the following two properties hold [3]:

1. If $x(t)$ is a solution, then $c \times x(t)$ is also a solution, where c is a constant.
2. If $x_1(t)$ and $x_2(t)$ are solutions, then $x_1 + x_2$ is a solution.

Equations 8.3.1 and 8.3.2 are linear, whereas Equations 8.3.3 is nonlinear differential equation.

8.3.2.3. Mass Balance around a Tank

There are several ways of obtaining a model and deduction based on the fundamental physical laws is one of them. Using the law of conservation of mass, mass of a liquid in a tank where a single stream of the liquid is entering and another single stream is leaving can be modeled as following [4]:

$$\text{Input} - \text{Output} = \text{Accumulation}$$

For a short time period of Δt ,

$$\begin{aligned} \text{Input} &= F \times \rho_{in} \times \Delta t \\ \text{Output} &= F' \times \rho_{out} \times \Delta t \\ \text{Accumulation} &= (\rho_{in} \times V \Big|_{t+\Delta t}) - (\rho_{out} \times V \Big|_t) \end{aligned}$$

where F is the inlet flow rate and F' is the outlet flow rate,

V is the volume that the liquid occupies in the tank at a certain time.

ρ is the density of the fluid (subscripts *in* and *out* denote the inlet and outlet flows),

$$F \times \rho_{in} \times \Delta t - F' \times \rho_{out} \times \Delta t = (\rho \times V \Big|_{t+\Delta t}) - (\rho \times V \Big|_t) \quad (8.3.6)$$

Dividing all terms by Δt and letting $\Delta t \rightarrow 0$:

$$F \times \rho_{in} - F' \times \rho_{out} = \frac{d(\rho V)}{dt} \quad (8.3.7)$$

If the fluid is assumed to be incompressible, ρ is constant, therefore all density terms cancel out. Furthermore, $V = A \times h$ (assuming the tank is a rectangular prism) and A (base area) is constant:

$$F - F' = A \frac{dh}{dt} \quad (8.3.8)$$

where h is the liquid level in the tank.

The outlet flow rate, F' may be maintained by the help of a pump to keep it constant. However, if no pump is used, outlet flow rate depends on the water height, gravity, the friction factor of the outlet pipe, etc. To account for the non-time dependent factors (like gravity and friction) an empirical constant R may be used, so the outlet flow rate may be modeled as following [5]:

a. A linear model where $F' \approx h$

$$F - \frac{h}{R} = A \frac{dh}{dt} \quad (8.3.9)$$

b. A nonlinear model where $F' \approx h^{0.5}$

$$F - \frac{h^{0.5}}{R} = A \frac{dh}{dt} \quad (8.3.10)$$

8.3.3. Apparatus

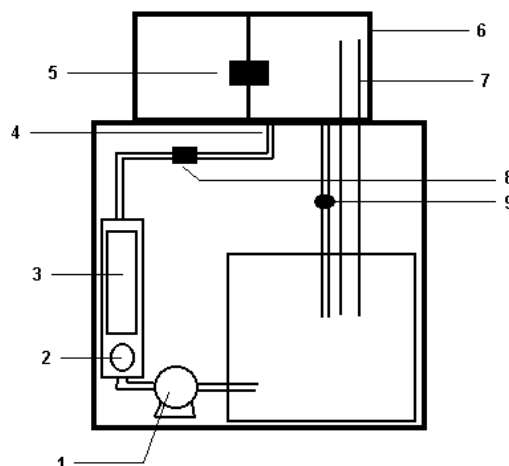


Figure 8.3.1. Process module.

- | | |
|----------------------------|---------------------------------|
| 1. Centrifugal pump | 6. Water tank |
| 2. Manual control valve | 7. Overflow |
| 3. Variable area flowmeter | 8. Motorized flow control valve |
| 4. Diffuser | 9. Drain valve |
| 5. Sealing stopper | |

8.3.4. Experimental Procedure

1. Adjust the flow rate of water to 0.9 L/min and 1 L/min, respectively. Record the steady state heights reached at these flow rates.
2. Starting from different initial water level heights, record the dynamic behavior of the water level at constant time intervals.
3. Do not forget to measure the length and width of the water tank.

8.3.5. Report Objectives

1. From the steady state heights, find R (resistance) values for linear and nonlinear models using **linear regression**. Discuss whether a difference in R values for different flow rates is plausible. Which model seems a better choice?
2. Using these R values, simulate these two models for the flow rates you performed in the experiment using **numerical integration** (please use ODE solver in MATLAB).
3. Show the simulation results and experimental data for the same flow rates on the same graph. Discuss which model fits better to the experimental behavior using root mean squared error (RMSE). Which parts of the dynamic response are better modeled? Why do you think it is so? How can you improve the models? Give recommendations.

8.3.6. References

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