ChE 203 - Physicochemical Systems Laboratory

EXPERIMENT 5: VELOCITY OF MOLECULES AND THE MAXWELL-BOLTZMANN DISTRIBUTION FUNCTION

Before the experiment: *Read the booklet carefully. Be aware of the safety issues.*

Object

To measure the velocity distribution of the 'model gas' and compare the result to theoretical behavior as described by the Maxwell-Boltzmann equation.

Theory

The Maxwell-Boltzmann distribution is a probability distribution with applications in physics and chemistry. The temperature of any (massive) physical system is the result of the motions of the molecules and atoms which make up the system. These particles have a range of different velocities, and the velocity of any single particle constantly changes due to collisions with other particles. However, the fraction of a large number of particles within a particular velocity range is nearly constant. The Maxwell distribution of velocities specifies this fraction, for any velocity range, as a function of the temperature of the system.

The equation for the kinetic energy of the molecules of an ideal gas along with the pressure of an ideal gas described by kinetic theory are combined with the ideal gas law to obtain

$$\bar{c} = \left(\frac{3 \times R \times T}{M}\right)^{1/2} \quad (1)$$

or

$$\bar{c} = \left(\frac{3 \times k \times T}{m}\right)^{1/2} \quad (2)$$

m Mass of the molecule

c Average velocity of the molecule

k Boltzmann constant

The direct determination of the velocity of a certain molecule is impossible because collisions with other molecules cause it to change incessantly. For a great number of molecules, a distribution function for molecular velocities can be derived by means of statistical methods. This was done by Maxwell and Boltzmann with the following result:

$$\frac{dN}{N} = \sqrt{\frac{2}{\pi}} \times \left(\frac{m}{k \times T}\right)^{3/2} \times c^2 \times e^{-\left(\frac{m \times c^2}{2 \times k \times T}\right)} \times dc \quad (3)$$

This equation describes the probability that the velocity of a molecule is within the interval {c,c+dc}. For the velocity at the maximum cw of the curve (velocity with highest probability) the following relation can be derived:

$$c_w = \left(\frac{2 \times k \times T}{m}\right)^{1/2} \quad (4)$$

Introducing of equation (3) into equation (4) leads to

$$\frac{dN}{N} = \frac{4}{\sqrt{\pi}} \times \left(\frac{1}{c_w^2}\right)^{3/2} \times c^2 \times e^{-\left(\frac{c^2}{c_w^2}\right)} \quad (5)$$

In the model experiment with glass balls, the velocity of the balls can be determined from the distance thrown s:

$$c = s\sqrt{\frac{g}{2h}} \quad (6)$$

g Acceleration at the earth surface (9.81 m/s^2)

h Height difference between outlet and receiver

Now the experimental results (number of balls per distance thrown interval) can be displayed graphically in the form

$$\frac{1}{\sum N_i} \times \frac{N_i}{\Delta c} = f(c) \quad (4)$$

N_i Number of balls in the interval i, i=1…24

Δc Velocity interval corresponding to Δs=1 cm (0.078 m/s)

Experimental Work

Apparatus and Chemicals: Kinetic gas theory apparatus, Receiver with recording chamber, Power supply, Precision balance, Digital stroboscope, Stopwatch, Tripod base, Glass beaker, Glass beads.

Procedure: Receiver is fitted with recording chamber to the apparatus. The average weight of one glass ball is determined by weighing out a known number of balls (e.g. 100). Following this, the average number of glass balls expelled from the apparatus during 1 minute is determined. To do this, the apparatus is filled with 400 glass balls and set to the following conditions:

- height of the upper piston: 6 cm
- oscillator frequency: 50 s^{-1} (controlled by the voltage and the stroboscope).

The outlet is opened for 1 minute and the number of balls expelled is determined by weighing them. The apparatus is refilled with these balls. The average number of balls expelled per minute is calculated and this number of glass balls is filled into four glass beakers. The following apparatus settings are made:

- height difference between outlet and receiver: 8 cm
- number of balls: 400
- oscillator frequency: 50 s^{-1}

When the frequency is stable, the outlet is opened for 5 minutes. After each minute, balls from one of the beakers are poured into the apparatus to maintain a constant ‘particle density’. The number of glass balls in each of the 24 compartments of the receiver is determined by weighing.

Safety Issues: Although water or any other chemicals are not used in this experiment, wearing gloves and lab coats is a must to prevent from any chemicals that is used at other experiments.

Calculations

1. Derive eqn 1 using kinetic theory.
2. Calculate c_w by taking m = ΣN_i x (mass of a hydrogen atom).
3. Plot $\frac{1}{\Sigma N_i} \times \frac{N_i}{\Delta c}$ vs. c to obtain the velocity distribution.
4. Read the experimental c_w value from the graph obtained from Q.3 and compare with the value obtained in Q.2.